# Matrix

Page No: | Date:

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Q.1] Construct a matrix $A = (a_{ij})_{3\times2}$ whose elements $a_{ij}$ is given by $a_{ij} = \frac{(i-j)^2}{5-i}$

$\Rightarrow$ Let's matrix of $A = (a_{ij})_{3\times2}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$a_{11} = \frac{(1-1)^2}{5-1} = \frac{(0)^2}{4} = 0$$
$$a_{12} = \frac{(1-2)^2}{5-1} = \frac{(-1)^2}{4} = \frac{1}{4}$$
$$a_{21} = \frac{(2-1)^2}{5-2} = \frac{(1)^2}{3} = \frac{1}{3}$$
$$a_{22} = \frac{(2-2)^2}{5-2} = \frac{(0)^2}{3} = 0$$
$$a_{31} = \frac{(3-1)^2}{5-3} = \frac{(2)^2}{2} = 2$$
$$a_{32} = \frac{(3-2)^2}{5-2} = \frac{(1)^2}{2} = \frac{1}{3}$$

$$\therefore A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{3} \end{bmatrix}$$

Q.2] find k if matrix A is a singular matrix where

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{vmatrix}$$

$$4(k-9) - 3(7-10) + 1(63-10k) = 0$$
$$4k - 36 + 12 + 63 - 10k = 0$$
$$-6k + 63 + 12 - 36 = 0$$
$$-6k + 39 = 0$$
$$6k = 39$$
$$k = \frac{39}{6}$$
$$k = \frac{13}{2}$$

Q. 3] If $A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 7 & -8 \\ 0 & -6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix}$ then find matrix 'C' such that A + B + C is zero matrix.

→ Let's $A + B + C = 0$

$A + B = -C$

$$-C = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 7 & -8 \\ 0 & -6 & 1 \end{bmatrix} + \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix}$$

$$-C = \begin{bmatrix} 1+9 & 2-1 & -3+2 \\ -3-4 & 7+2 & -8+5 \\ 0+4 & -6+0 & 1-3 \end{bmatrix}$$

$$-C = \begin{bmatrix} 10 & 1 & -1 \\ -7 & 9 & -3 \\ 4 & -6 & -2 \end{bmatrix}$$

$$C = \begin{bmatrix} -10 & -1 & 1 \\ 7 & -9 & 3 \\ -4 & 6 & 2 \end{bmatrix}$$

Q.4] If $A = \begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix}$

find matrix '$x$' such that $3A - 4B + 5X = C$

$\Rightarrow$ Let's $3A - 4B + 5x = C$

$$3\left[ \quad 5x = C + 4B - 3A \quad \right]$$

$$5x = \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + 4\begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}$$

$$5x = \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ 16 & 8 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 3 & -6 \\ 9 & -15 \\ -18 & 0 \end{bmatrix}$$

$$5x = \begin{bmatrix} 2-4-3 & 4-8+6 \\ -1+16-9 & -4+8+15 \\ -3+4+18 & 6+20-0 \end{bmatrix}$$

$$5x = \begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix}$$

$$x = \frac{1}{5}\begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix} \qquad \therefore x = \begin{bmatrix} -1 & \frac{2}{5} \\ \frac{6}{5} & \frac{19}{5} \\ \frac{19}{5} & \frac{26}{5} \end{bmatrix}$$

Q.5] Find, $x, y, z$ and $a, b, c$ if $\begin{bmatrix} a & 5i & x \\ y & b & z \\ \frac{3}{2} & -\sqrt{2} & c \end{bmatrix}$ is Skew-Symmetric matrix.

$\longrightarrow$ $A = \begin{bmatrix} a & 5i & x \\ y & b & z \\ \frac{3}{2} & -\sqrt{2} & c \end{bmatrix}$ is Skew Symmetric matrix

By the following def$^n$. diagonal of Skew Symmetric matrix is zero.

$$a_{ij} = -a_{ji}.$$

$$\therefore \quad a = 0 \quad b = 0 \quad c = 0$$

$$x = -\frac{3}{2} \quad y = -5i \quad z = \sqrt{2}$$

Q.6] Find $x, y, z$ if

(i) $\begin{bmatrix} 2x+1 & -1 \\ 3 & 4y \end{bmatrix} + \begin{bmatrix} -1 & 6 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 6 & 12 \end{bmatrix}$

$$\begin{bmatrix} 2x+1 & -1 \\ 3 & 4y \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} -1 & 6 \\ 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2x+1 & -1 \\ 3 & 4y \end{bmatrix} = \begin{bmatrix} 4+1 & 5-6 \\ 6-3 & 12-0 \end{bmatrix}$$

$$\begin{bmatrix} 2x+1 & -1 \\ 3 & 4y \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & 12 \end{bmatrix}$$

By equaillity of matix.

Page No :
Date

$$2x + 1 = 5 \qquad 4y = 12$$
$$2x = 5 - 1 \qquad y = \frac{12}{4}$$
$$x = \frac{4}{2} \qquad \boxed{y = 3}$$
$$\boxed{x = 2}$$

(ii) $$\left\{4\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 2 & -3 & 4 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \left\{4\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 2 & -3 & 4 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left\{\begin{bmatrix} 8 & -4 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 2 & -3 & 4 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 8-1 & -4-2 & 12+1 \\ 4-2 & 0+3 & 8-4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 7 & -6 & 13 \\ 2 & 3 & 4 \end{bmatrix}_{2\times 3}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 14+6+13 \\ 4-3+4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 33 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$ By equolility of matix.

$$x = 33$$
$$y = 5.$$

(iii) $(5A-3B)C = X$ where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -2 & 3 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left(5\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ -2 & 3 \\ 3 & 1 \end{bmatrix}\right)\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \\ 5 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ -6 & 9 \\ 9 & 3 \end{bmatrix}\right)\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\underset{3\times 2}{\begin{bmatrix} 2 & -6 \\ 6 & -4 \\ -4 & 2 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 4-6 \\ 12-4 \\ -8+2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ By equallity of matrix

$x = -2$

$y = 8$

$z = -6$

Q.7] $A = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$. Show that $A^2$ is null matrix.

$A^2 = A.A$

$$A^2 = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4-4 & 8-8 \\ -2+2 & -4+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ $\therefore A^2$ is null matrix.

Q.8] If $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $A^2 - 7A + KI = 0$ then find K.

$\Rightarrow$ Let's $A^2 - 7A + KI = 0$

$$\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} - 7 \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} + K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0.$$

$$\begin{bmatrix} 16+2 & 8+6 \\ 4+3 & 2+9 \end{bmatrix} - \begin{bmatrix} 28 & 14 \\ 7 & 21 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} = 0$$

$$\begin{bmatrix} 18-28 & 14-14 \\ 7-0 & 11-21 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} = 0$$

$$\begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} -K & 0 \\ 0 & -K \end{bmatrix}$$ By equallity of matrix

$$\boxed{\therefore K = 10}$$

Page No
Date

Q.9] If $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find $|AB|$

$$\Rightarrow \quad A.B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+3 & 2+4 \\ 2+6 & 4+8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}$$

Q.10] If $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix}$ and if $(A+B)^2 = A^2+B^2$

Find values of a and b.

$$(A+B)^2 = A^2+B^2$$
$$(A+B)(A+B) = A^2+B^2$$
$$\cancel{A^2}+AB+BA+\cancel{B^2} = \cancel{A^2}+\cancel{B^2}$$

$$AB = -BA$$

$$\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} = -\begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

By equality

$$\begin{bmatrix} 2+(-2) & a+2b \\ -2+2 & -a-2b \end{bmatrix} = -\begin{bmatrix} 2-a & 4-2a \\ -1-b & -2-2b \end{bmatrix}$$

$-2+a=0$

$a=2$

$$\begin{bmatrix} 0 & a+2b \\ 0 & -a-2b \end{bmatrix} = \begin{bmatrix} -2+a & -4+2a \\ +1+b & 2+2b \end{bmatrix}$$

$1+b=0$

$b=-1$