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 Std - XIth Subject - Maths - 1

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Q1)

a) $3+i$

→ complex conjugate of $3+i$ is $3-i$

b) $3-i$

→ complex conjugate of $3-i$ is $3+i$

c) $-\sqrt{5}i - \sqrt{7}i$

→ complex conjugate of $-\sqrt{5}i - \sqrt{7}i$ is $-\sqrt{5}i + \sqrt{7}i$

d) $-\sqrt{-5}$

→ complex conjugate of $-\sqrt{-5}$ is $\sqrt{-5}$

Q2

a) i^{35}

$$\begin{aligned} & (i^2)^6 (i^2) (i^2) (i) \\ & (-1)^6 (-1) (-1) i \\ & 1 \times 1 \times i \\ & i \end{aligned}$$

$$\begin{aligned} & (i^2)^{17} (i) \\ & (-1)^{17} (i) \\ & -i \end{aligned}$$

b) i^{888}

$$(i^2)^{444} (-1)^{444} = 1$$

$$\begin{aligned}
 3) \quad & i^{93} \\
 & = i^{92} (i) = (-1)^{58} (-1)^{46} (i) \\
 & = (i^2)^{46} (i) = (-1)^{46} (i) = -i = i
 \end{aligned}$$

$$4) \quad i^{30} + i^{40} + i^{50} + i^{60}$$

$$\begin{aligned}
 & (i^2)^{15} + (i^2)^{20} + (i^2)^{25} + (i^2)^{30} \\
 & (-1)^{15} + (-1)^{20} + (-1)^{25} + (-1)^{30} \\
 & -1 + 1 - 1 + 1 = 0
 \end{aligned}$$

Q3

$$\begin{aligned}
 a) \quad & (2+3i)(2-3i) \\
 & 4 - 6i + 6i - 9i^2 \\
 & 4 - 9(-1)^2 \\
 & 4 + 9 = 13 + 0i \\
 & a = 13 \quad b = 0i
 \end{aligned}$$

$$b) \quad \frac{i(4+3i)}{(1-i)}$$

$$\rightarrow \frac{i(4+3i)}{(1-i)} \times \frac{(1+i)}{(1+i)}$$

$$\frac{(4i+3i^2)(1+i)}{(1)^2 - (i)^2}$$

$$\frac{4i + 4i^2 + 3i^2 + 3i^2 i}{1+1} = \frac{4i - 4 - 3 - 3i}{2} = \frac{i^2 - 7}{2}$$

$$\frac{7}{2} - \frac{i}{2}$$

$$\frac{7}{2} - \left(\frac{1}{2}\right)i$$

$$a = \frac{7}{2} \quad b = -\frac{1}{2}$$

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$$7 + 24i$$

$$\sqrt{7 + 24i} = (a + bi)^2$$

$$7 + 24i = a^2 + 2abi + (bi)^2$$

$$7 + 24i = (a^2 - b^2) + 2abi$$

equating real and imaginary we get

$$7 + a^2 - b^2 = 7 \quad 2abi = 24i$$

$$b = \frac{24}{2} = \frac{12}{a}$$

$$a^2 - b^2 = 7$$

$$a^2 - \left(\frac{12}{a}\right)^2 = 7$$

$$a^2 - \frac{144}{a^2} = 7$$

$$a^4 - 144 = 7a^2$$

$$a^4 - 7a^2 - 144 = 0$$

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$$a^4 - 16a^2 + 9a^2 - 144 = 0$$

$$a^2(a^2 - 16) + 9(a^2 - 16)$$

$$(a^2 + 9) = 0 \quad (a^2 - 16) = 0$$

$$a^2 = -9$$

$$a^2 = 16$$

$$a = \pm 4$$

$$\text{when } a = 4 \quad b = \frac{12}{4} = 3$$

$$\text{when } a = -4 \quad b = \frac{-12}{-4} = 3$$

The square roots of $7 + 24i$
 $4 + 3i$ or $-4 - 3i$

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$$3x^2 - 7x + 5 = 0$$

$$a = 3 \quad b = -7 \quad c = 5$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 3 \times 5}}{2 \times 3}$$

$$\frac{7 \pm \sqrt{49 - 60}}{6}$$

$$\frac{7 \pm \sqrt{-11}}{6}$$

$$\frac{7 + \sqrt{11(-1)}}{6} = \frac{7 + \sqrt{11}}{6} \quad \text{and} \quad \frac{7 - \sqrt{11}}{6}$$

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a)

~~w is a complex unity of cube root of unity~~

~~$$w^3 = 1 \text{ and } 1 + w + w^2 = 0$$~~

a) b)

~~$$(2 - w)(2 - w^2)$$~~

~~$$4 - 2w^2 - 2w + w^3$$~~

~~$$4 - 2(w^2 + w) + 1$$~~

~~$$4 - 2(-1) + 1$$~~

~~$$4 + 2 + 1$$~~

~~$$2 + 3 = 7 = \text{RHS}$$~~

$$b) (a + b) + (aw + bw^2) + (aw^2 + bw) = 0$$

~~$$(a + b)$$~~

~~$$a + b + aw + bw^2 + aw^2 + bw$$~~

~~$$(a + aw + aw^2)(b + bw + bw^2)$$~~

~~$$a(1 + w + w^2) + b(1 + w + w^2)$$~~

~~$$a(w^3) + b(w^3)$$~~

~~$$a(1) + b(1)$$~~

~~$$a(0) + b(0) = 0 = \text{RHS}$$~~