

Properties of determinants

Property 1: The value of determinant remains unchanged if its rows are turned into columns and columns are turned into rows.

$$\text{ex: } \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 0 & 2 & 1 \end{vmatrix} = -17 \quad \text{--- (1)}$$

also,

$$\begin{vmatrix} 1 & 3 & 0 \\ 2 & -1 & 2 \\ -1 & 2 & 1 \end{vmatrix} = -17 \quad \text{--- (2)}$$

$$\text{i)} \begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(-1-4) - 2(3-0) + 1(6-0) \\ = -5 - 6 + 6 \\ = -17$$

$$\text{ii)} \begin{vmatrix} 1 & 3 & 0 \\ 2 & -1 & 2 \\ -1 & 2 & 1 \end{vmatrix} = 1 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - 3 \begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} + 0 \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 1(-1-4) - 3(2+2) + 0 \\ = -5 - 12 \\ = -17$$

Date _____
Page _____

• property 2: If any two rows (or columns) of a determinant are interchanged then the value of determinant change only is sign.

$$\text{Ex : i)} \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 0 & 2 & 1 \end{vmatrix}$$

$$\begin{aligned} &= 1(-1-4) - 2(3-0) - 1(6-0) \\ &= -5 - 6 - 6 \\ &= -17 \end{aligned}$$

$$\text{ii)} \begin{vmatrix} 2 & 1 & -1 \\ -1 & 3 & 2 \\ 2 & 0 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$$

$$\begin{aligned} &= 2(3-0) - 1(-1-4) - 1(0-6) \\ &= 6 + 5 + 6 \\ &= 17 \end{aligned}$$

Property 3: If any two rows (or column) of a determinant are identical then the value of the determinant is zero.

$$\text{Ex : } \begin{vmatrix} 1 & -1 & 2 \\ 1 & -1 & 2 \\ 0 & 1 & 2 \end{vmatrix} = 1 \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 1(-2-2) - (-1)(2-0) + 2(1-0)$$

$$= -4 + 2 + 2 = -4 + 4 = \underline{\underline{0}}$$

Property 4: If each element of a row (or column) of a determinant is multiplied by a constant k then the value of the new determinant is k times the value of the original determinant.

$$\text{ex : i)} \begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 0 - 2(-3-2) + 1(6+1)$$

$$= 10 + 7$$

$$= 17$$

$$\text{ii)} \begin{vmatrix} 0 & 2 & 1 \\ 3 \times 3 & -1 \times 3 & 2 \times 3 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 2 & 1 \\ 9 & -3 & 6 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 0 - 2(-9 - 6) + 1(18 + 3)$$

$$= 30 + 21$$

$$= 51$$

property 5: If ^{all} each element of a row (or column) is expressed as the sum of two numbers then the determinant can be expressed as sum of two determinants.

$$\text{Ex: } \begin{vmatrix} 3 & 2 & 1 \\ 6 & 2 & 4 \\ 2 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 2+1 & 1+1 & 0+1 \\ 6 & 2 & 4 \\ 2 & 1 & 6 \end{vmatrix}$$

$$\text{or } = \begin{vmatrix} 3 & 2 & 1 \\ 3+3 & 1+1 & 3+1 \\ 2 & 1 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 2 & 1 \\ 3 & 1 & 3 \\ 2 & 1 & 6 \end{vmatrix} + \begin{vmatrix} 3 & 2 & 1 \\ 3 & 1 & 1 \\ 2 & 1 & 6 \end{vmatrix}$$

Property 6: If a constant multiple of all elements of any row (or column) is added to the corresponding element of any other row (or column) then the value of new determinant so obtained is the same as that of the original determinant.

Ex: $B = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 1 & 2 & 1 \end{vmatrix} \quad R_3 = 3R_1 + R_3$

$$B_1 = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 4 & 8 & 10 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 0 & -2 & -1 & 0 & 3 & -1 & 2 \\ & & 8 & 10 & 4 & 10 & & 4 & 8 \end{vmatrix}$$

$$\begin{aligned} &= 1(20-0) - 2(-10-0) + 3(-8-8) \\ &= 20 + 20 - 48 \\ &= -8 \end{aligned}$$

$$\therefore C_2 = 5C_1 + C_2$$

$$B_2 = \begin{vmatrix} 1 & 7 & 3 \\ -1 & -3 & 0 \\ 4 & 7 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 0 \\ & 7 & 1 \end{vmatrix} - 7 \begin{vmatrix} -1 & 0 \\ & 1 \end{vmatrix} + 3 \begin{vmatrix} -1 & -3 \\ & 7 \end{vmatrix}$$

$$= 1(-3-0) - 7(-1-0) + 3(-7-(-3))$$

$$\begin{aligned} &= -3 + 7 - 12 \\ &= -8 \end{aligned}$$

Property 7: (Triangle property): If all the elements of a determinant above or below the diagonal are zero then the value of the determinant is equal to the product of its diagonal elements.

ex :
$$A = \begin{vmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 2 & 2 \end{vmatrix} = 3 \times 4 \times 2 = 24$$